

Calculators, Mobile-telephones and Pagers ARE NOT ALLOWED.

Answer ALL of the following questions.

All items are weighted equally at 4 points each.

1. (a) Use the definition of the limit to show that  $\lim_{x \rightarrow 2} (3x - 1) = 5$ .

(b) Evaluate each of the following limits, if it exists

(i)  $\lim_{x \rightarrow 1} \frac{\sqrt{x+2} - \sqrt{3}}{x-1}$

(ii)  $\lim_{x \rightarrow 0} \frac{\sin^2 x + x^2}{x \sin 3x}$

2. (a) Use the definition of the derivative to find  $f'(a)$  where  $f(x) = x^3$ .

(b) Find the points at which the graph of the function  $f(x) = (x-1)^{\frac{1}{5}} + (x-2)^{\frac{2}{3}}$  has a vertical tangent line or a cusp.

(c) Find the vertical and horizontal asymptotes, if any, of  $f(x) = \frac{x\sqrt{1+x^2}}{(1-x)|x|}$ .

(a) If  $f(x) = \cos^2(\sqrt{2x^3+3})$ , then find  $f'(x)$ .

(b) Find the equation of the normal line to the graph of  $f(x) = 1 + \frac{x^2+x}{\sec(2x)}$ , at  $x=0$ .

$$f(x) = \begin{cases} (k-2)x - \frac{x^3}{3}, & \text{if } x < 0 \\ x^2 \sin \frac{1}{x} & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) Show that  $f$  is continuous at  $x=0$  for all  $k \in \mathbb{R}$ .

(b) Find the value(s) of  $k$  for which  $f$  is differentiable at  $x=0$ .

Good Luck